

# **Rolling the Dice on the Delayed-Choice Quantum Eraser**

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## ***Abstract***

This paper offers an alternative interpretation of the delayed-choice quantum eraser using the analogy of tossing two quantumly entangled dice. This interpretation duplicates the symmetric and antisymmetric data plots of the original paper by Kim et al. and provides a deeper insight into the statistical nature of the experiment. In particular, it proposes a stochastic explanation for why the original coincidence-detection plots do not go to zero where theory predicts they should be zero. In so doing, it also presents a simple metaphor of the experiment as a collection of random dice throws.

## ***Introduction***

It was Albert Einstein himself who famously signaled his skepticism of quantum mechanics with the quote “God does not play dice with the world.” With this one declaration he tossed aside the uncomfortable indeterminacy of the quantum world for the more familiar determinism of classical physics. Einstein never did make peace with quantum mechanics, and he repeated his dice metaphor throughout his life. But the quantum description of physical reality prevailed, and it now forms our most complete and accurate understanding of the universe.

To be clear, quantum mechanics is not for the faint-of-heart. It’s a theory that flies in the face of our classical intuitions. But there are two experiments in particular that convincingly demonstrate two of the most bizarre yet fundamental effects of quantum behavior: superposition and entanglement. The two experiments are Young’s renowned double-slit experiment (superposition) and the now famous delayed-choice quantum eraser (superposition and entanglement), which is a 21st-Century update of Young’s experiment.

This paper examines the delayed-choice quantum eraser experiment of Kim et al. by equating it with the random throws of two dice. The metaphor not only seems to explain the data plots of the experiment, but this interpretation also offers a deeper understanding of its random, stochastic nature.

## ***The experimental setup of Kim et al.***

I already have given a fuller description of the physics behind the delayed-choice quantum eraser, which I published in March 2025 on my website. [\[1\]](#) Figure 1 of that paper (and this one) shows the basic experimental setup (see Figure 1).

At the bottom of Figure 1, UV light (351.1-nm) from an argon-ion laser (blue arrows) irradiates a double-slit, just like in Young’s classic experiment. However, the similarity to Young’s experiment quickly disappears next, because the wavefunction of each coherent UV photon that passes through the two slits is promptly used to pump two regions of a  $\beta$ -Barium Borate (BBO) crystal (labeled 1 and 2 in the figure).

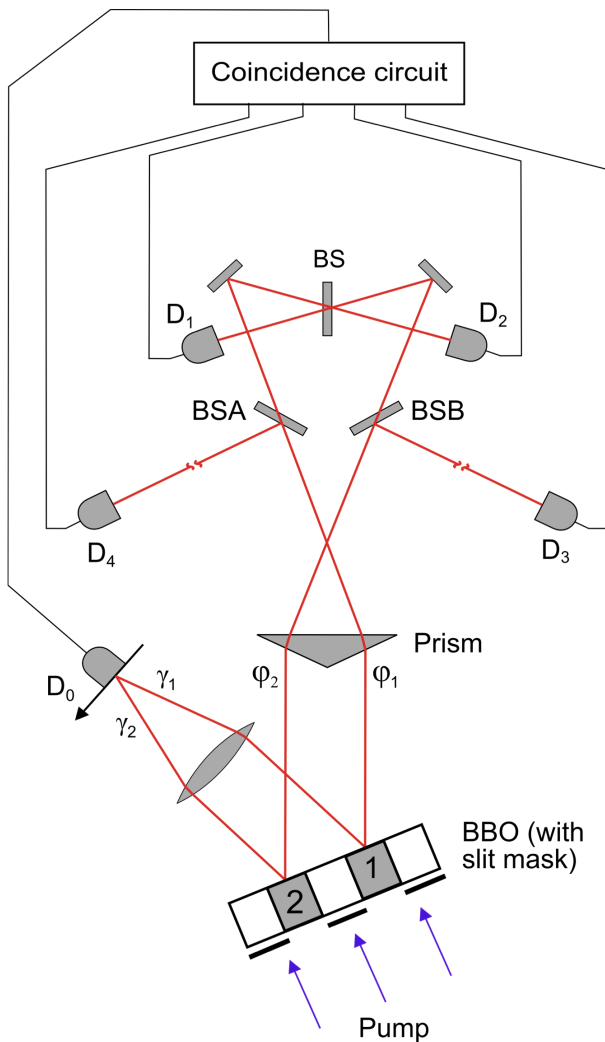


Figure 1

A tiny minority of pump photons ( $1$  in  $10^6$ ) are transformed within the crystal into two entangled photons (signal  $\gamma$  and idler  $\phi$ ), each having twice the wavelength (702.2 nm) and therefore half the frequency or energy.

Signal photons ( $\gamma$ ) emitted from the two BBO regions are directed off to the left in Figure 1 where they encounter a lens that focuses them onto the scanning detector  $D_0$ . This detector looks for any intensity variations that might appear along the focal plane where the two beams intersect.

Idler photons ( $\phi$ ) exit the BBO to a separate area of the setup in Figure 1, where they encounter a prism, three beamsplitters (BSA, BSB, BS), two mirrors, and four detectors ( $D_1$ ,  $D_2$ ,  $D_3$ ,  $D_4$ ). Optical path lengths from the BBO to these four detectors  $D_j$  ( $j = 1-4$ ) are all made equal, but they also are intentionally made longer than the optical path length

from the BBO to  $D_0$ . This creates a time delay between the detections of each idler photon and its entangled signal-photon twin. In the original experiment, this delay amounts to “at least 8 ns.”

Each of the  $D_j$  detectors is also electronically linked with  $D_0$  for coincidence detection (see Figure 1 again). Since each signal-idler pair is created simultaneously, coincidence detections will generate a ledger containing four subsets of synchronized event data. Each detection event at  $D_0$  will therefore have a corresponding detection of its entangled twin 8 ns later at one of the four  $D_j$  detectors.

### ***The quantum-dice matrix***

Here's how we might describe this experiment as the dice rolls of two four-sided dice: Each signal photon ( $\gamma$ ) randomly encountered by the scanning detector  $D_0$  *potentially* belongs to one of four states in a statistical ensemble of mixed quantum states. The four states are: BBO regions 1 and 2 acting together symmetrically ( $\Psi_1$ ); BBO regions 1 and 2 acting together antisymmetrically ( $\Psi_2$ ); BBO region 2 acting alone ( $\Psi_3$ ); and BBO region 1 acting alone ( $\Psi_4$ ). *But, only the joint detections of signal detector  $D_0$  and idler detector  $D_j$  ( $j = 1-4$ ) can decide which state the photon belongs to ( $\Psi_1, \Psi_2, \Psi_3$ , or  $\Psi_4$ ).* Meanwhile, each entangled idler twin ( $\phi$ ) is randomly intercepted by either  $D_1, D_2, D_3$ , or  $D_4$ . (There are important exceptions to this random detector distribution that will be introduced soon.)

The random detections at  $D_0$  might therefore be modeled as the random outcomes of a four-sided die. The random detections 8 ns later of each entangled idler twin at  $D_j$  could be thought of as the arbitrary outcomes of another four-sided die. Coincidence detections record the combinations of these correlated detections of the signal-idler pairs.

As such, the delayed-choice quantum eraser seems like a possible candidate for applying the statistical mathematics of tossing two four-sided dice. We start by building a matrix of all possible combinations of coincidence detections, which would look like this:

$$\begin{pmatrix} \Psi_1 D_1 & \Psi_2 D_1 & \Psi_3 D_1 & \Psi_4 D_1 \\ \Psi_1 D_2 & \Psi_2 D_2 & \Psi_3 D_2 & \Psi_4 D_2 \\ \Psi_1 D_3 & \Psi_2 D_3 & \Psi_3 D_3 & \Psi_4 D_3 \\ \Psi_1 D_4 & \Psi_2 D_4 & \Psi_3 D_4 & \Psi_4 D_4 \end{pmatrix}$$

Matrix 1

In Matrix 1,  $\Psi_i$  ( $i = 1-4$ ) symbolizes the four signal states *potentially* detected by  $D_0$ . The four idler detectors are denoted by rows  $D_j$  ( $j = 1-4$ ). So there are 16 combinations ( $4 \times 4$ ) represented. Each matrix row represents all of the joint-detection rates involving a particular  $D_j$  idler detector. Each column distributes—to all  $D_j$  idler detectors—the portion (calculated from theory) that  $\Psi_i$  would contribute to the total joint-detection rate measured at each  $D_0$  position.

### ***Applying the quantum-dice matrix to the experiment***

In a fair toss of two four-sided dice, the 16 possible outcomes are entirely independent of each other. But in the delayed-choice quantum eraser, the distribution of coincidence detections is more complex. We must, therefore, modify our matrix to match the experimental conditions.

First, there are four impossible combinations in the experiment. Specifically, the coincidence detections  $\Psi_1 D_2$  and  $\Psi_2 D_1$  cannot occur because of the sorting property of the Mach-Zehnder quantum eraser, whereby  $D_1$  registers only symmetric idler photons, and  $D_2$  registers only antisymmetric idler photons. Therefore,  $\Psi_2 D_1$ ,  $\Psi_1 D_2$  coincidences must be zero.  $\Psi_3 D_4$  joint detections also are impossible because  $D_4$  isn't even looking at  $\Psi_3$  wavefunctions coming only from BBO region 2, so it can't detect them (refer to Figure 1). The same goes for  $\Psi_4 D_3$ , since  $D_3$  isn't looking at BBO region 1, either. It follows that these four coincidence detections will always be zero in the matrix, like so:

$$\begin{pmatrix} \Psi_1 D_1 & 0 & \Psi_3 D_1 & \Psi_4 D_1 \\ 0 & \Psi_2 D_2 & \Psi_3 D_2 & \Psi_4 D_2 \\ \Psi_1 D_3 & \Psi_2 D_3 & \Psi_3 D_3 & 0 \\ \Psi_1 D_4 & \Psi_2 D_4 & 0 & \Psi_4 D_4 \end{pmatrix}$$

Matrix 2

The 16 combinations in Matrix 2 represent the possible random coincidence detections in the experiment. *It is important to note here that while the interaction probability of the idler photon at the four  $D_j$  detectors is still completely random with one-in-four odds, the detection rate of the entangled signal photons at detector  $D_0$  depends on the detector's position and the intensities of  $\Psi_i$  wavefunctions at that location. This alters the joint-detection rates of the signal-idler pairs accordingly, because the signal detections at  $D_0$  are paired one-for-one with their correlated random  $D_j$  idler detections. But any  $D_0$  detection made without a corresponding idler-twin detection is ignored (zeroed), as shown in Matrix 2.*

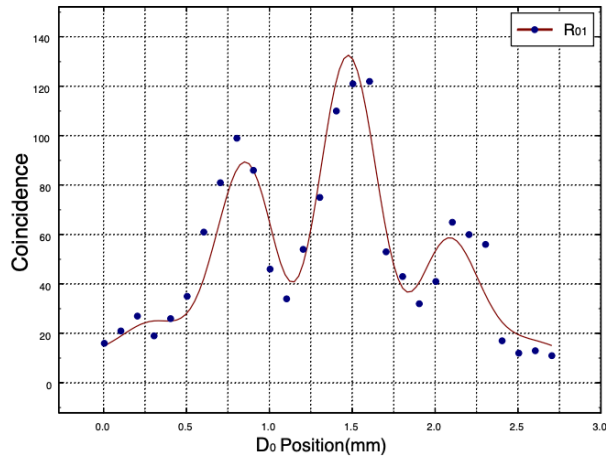


Figure 2 (from [2])

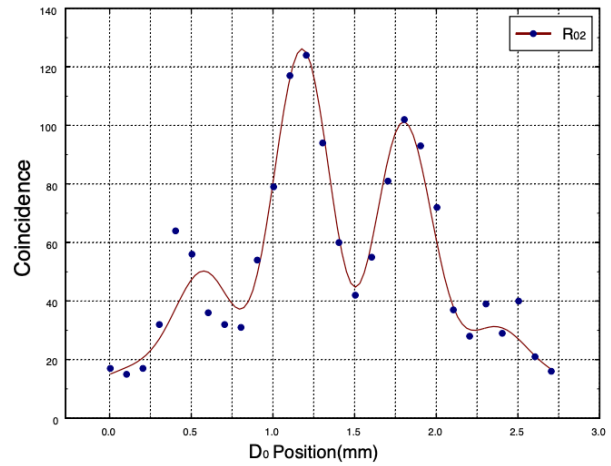


Figure 3 (from [2])

Notice, also, that coincidence detections involving  $D_1$  (top matrix row) are not only paired with  $\Psi_1$  signal-photon detections at  $D_0$ , but with the  $\Psi_3$  and  $\Psi_4$  photons as well. A similar effect can be seen in the other three rows of the matrix, too. Consequently, the data plots of coincidence detections should include all of these data. And, in fact, this is exactly what we see in the data plots presented in the original paper by Kim et al. (see Figures. 2 & 3). Their joint-detection curves ride above a conspicuous floor instead of going to zero where theory predicts. This key detail is neither explained nor acknowledged by the authors in the original paper. [2]

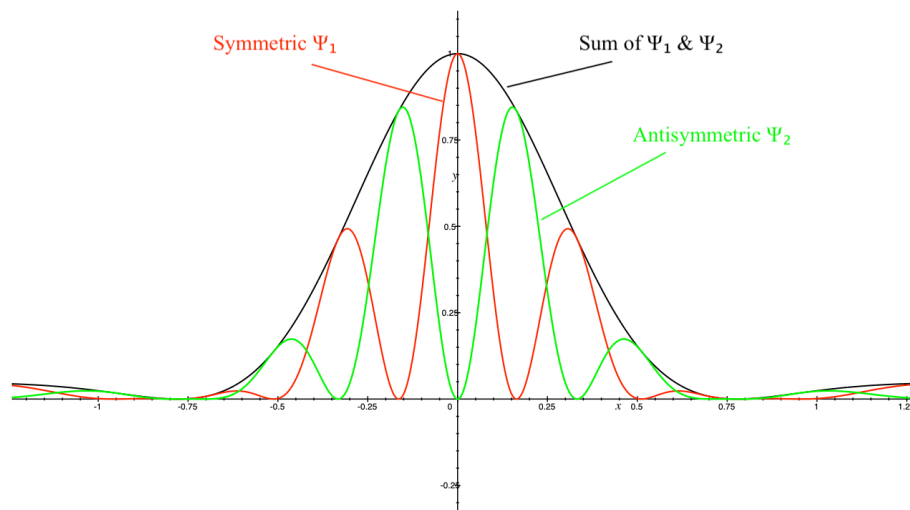


Figure 4

Standard theory predicts that the intensity (i.e. probability) plots of interfering symmetric and antisymmetric wavefunctions  $\Psi_1$  and  $\Psi_2$  (red and green plots in Figure 4 above) should periodically go to zero at specific locations in space. Clearly, this is not what we see in the symmetric and antisymmetric joint-detection plots of Figures 2 and 3. The disparity results from an accumulation of the other random joint detections shown in rows 1 and 2 of Matrix 2. These include  $\Psi_3 D_1$ ,  $\Psi_4 D_1$  in row 1 and  $\Psi_3 D_2$ ,  $\Psi_4 D_2$  in row 2; so, we should also include  $\Psi_3$  and  $\Psi_4$  probability waveforms in Figure 4, which I have done in Figure 5 (blue curves).

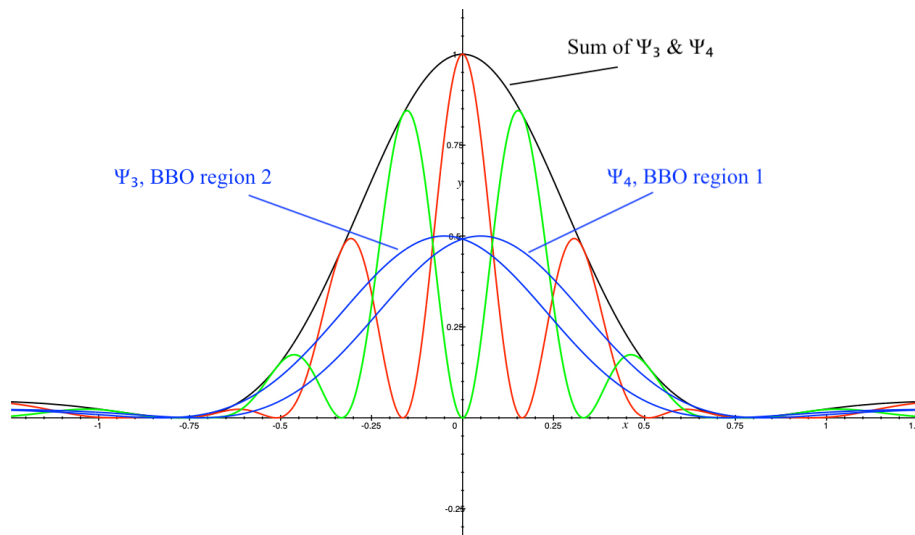


Figure 5

Now we can use the four theoretical intensities plotted in Figure 5 to calculate how much  $\Psi_3$  and  $\Psi_4$  contribute to the measured joint-detection rates plotted in Figures 2 and 3. [Even though the arbitrary axes of Figure 5 don't match those of Figures 2 and 3, we can still use them. Just multiply the y-axis values of Figure 5 by 100.]

In the  $D_0$  scanning plot of Figure 2, we see that its central maximum lies at the position of 1.5 mm, while the first minimum occurs at approximately 1.8 mm. We know from the corresponding red curve in Figure 5 that this minimum should be zero, but the lowest data point in Figure 2 shows a value of just over 30 at this minimum, so this must be the data floor level. The maximum, on the other hand, seems to be at about 125 or 130. Now, from the theoretical

plots of  $\Psi_3$ , and  $\Psi_4$  in Figure 5 (blue curves), the joint detections at  $x = 0$ , which corresponds to the  $D_0$  position of 1.5 mm in Figure 2, should each contribute a rate of about 50 ( $0.5 \times 100$ ).

For the  $D_0$  antisymmetric plot of Figure 3, the minimum occurs at the same position (1.5 mm) as the symmetric maximum does in Figure 2, as expected. Let's see what we get when we use numbers based on the theoretical plots of Figure 5 for the entries of Matrix 2.

With an assumed joint-detection floor level of 30, the measured maximum rate of 130 for  $\Psi_1 D_1$  in Figure 2 must actually be 100 ( $130 - 30$ ). We'll use this number for the theoretical maximum at the midpoint of 1.5 mm, which, again, corresponds to the y-axis ( $x = 0$ ) in Figure 5. At this  $D_0$  position, the Figure-5 rates for  $\Psi_1 D_1$ ,  $\Psi_2 D_1$ ,  $\Psi_3 D_1$  and  $\Psi_4 D_1$  in row 1 of Matrix 2 would be 100, 0, 50, 50. The second-row antisymmetric values of  $\Psi_1 D_2$ ,  $\Psi_2 D_2$ ,  $\Psi_3 D_2$ , and  $\Psi_4 D_2$  would be 0, 0, 50, 50. Row 3 is 100, 0, 50, 0, and row 4 is 100, 0, 0, 50. (See Matrix 3.)

$$\begin{pmatrix} \boxed{100} & 0 & 50 & 50 \\ 0 & \boxed{0} & 50 & 50 \\ 100 & 0 & \boxed{50} & 0 \\ 100 & 0 & 0 & \boxed{50} \end{pmatrix}$$

Matrix 3

The diagonal of Matrix 3 (red boxes), where the column and row indexes are equal ( $i = j$ ), represents the joint-detection rates of each signal wavefunction  $\Psi_i$  with its matching idler detection ( $\Psi_1 D_1$ ,  $\Psi_2 D_2$ ,  $\Psi_3 D_3$ ,  $\Psi_4 D_4$ ). The off-diagonal entries represent the contributions from all of the other possible joint detections. Each row denotes all of the joint-detection rates involving a single idler detector—for example  $D_1$  in the first row. *To find the combined joint-detection rates of  $\Psi_i$  wavefunctions that involve a single idler detector, we add the average value of the off-diagonal elements in that matrix row to the diagonal element of that same row.* In this way we account for the random “background” joint detections at the idler detector of interest. Also, bear in mind that each new  $D_0$  position generates a whole new matrix of joint-detection rates.

The joint-detection rate of the symmetric state in Figure 2 is represented by the  $D_1$  row of Matrix 3 (row 1). This consists of  $\Psi_1 D_1$  plus the average contributions from the three off-



diagonal joint detections ( $\Psi_2 D_1$ ,  $\Psi_3 D_1$  and  $\Psi_4 D_1$ ). Thus we have  $100 + (0 + 50 + 50)/3 = 133.33$ , which is pretty close to the measured coincidence rate of approximately 130 given in Figure 2 at a  $D_0$  position of  $x = 1.5$  mm. So the estimated data floor of 30 does seem to yield some agreement with experiment when calculating the theoretical values for Matrix 3 from Figure 5.

Row 2 of Matrix 3 gives us the antisymmetric  $D_2$  coincidence data for  $D_0$  at  $x = 1.5$  mm. This is  $0 + (0 + 50 + 50)/3 = 33.33$ , which is not too far off from the value of 40 given in Figure 3 at  $x = 1.5$  mm. However, it is virtually equal to the other antisymmetric minimum plotted around  $x = 0.75$  mm in Figure 3. Unlike the theoretical values of Figure 5, the actual experimental data plotted in Figures 2 and 3 are a little asymmetric and skewed.

The original paper includes a plot of  $D_0$ - $D_3$  joint-detections, too, but for some reason it does not yield as close a match with the detection rates predicted by Matrix 3. According to the matrix, joint-detection rates involving  $D_3$  (or  $D_4$ ) at a  $D_0$  position of 1.5 mm should be  $50 + (100 + 0 + 0)/3 = 83.33$ . But the original paper lists a joint-detection rate of about 118 for  $D_0$ - $D_3$  at this position. I cannot explain this discrepancy without knowing more about how this data was obtained and plotted, but the theoretical detection probability for  $\Psi_3$  (and  $\Psi_4$ ) at this midway position is half that of  $\Psi_1$ , for example. Therefore, the detection rate of  $\Psi_3$  signal photons at  $D_0$  also should be half for a detector of a given quantum efficiency. And this same rate difference must necessarily apply to the correlated idler twins arriving at  $D_j$  detectors 8 ns later. The  $D_0$ - $D_3$  coincidence plot in the original paper should therefore reflect this detection-rate difference.

I have applied the same quantum-dice matrix strategy to the data of Figures 2 and 3 at  $D_0$  positions other than the midpoint of 1.5 mm, and the results all come close to the experimental values. They even duplicate the decreasing floor level as  $D_0$  moves away from 1.5 mm, an effect that can readily be seen in the figures. The data floor is not flat. It bows upward in the middle.

### ***Review and conclusions***

This interpretation of the delayed-choice quantum eraser experiment began by applying the analogy of tossing two four-sided dice. But this simple metaphor of a game of chance had to be altered to fit the quantum conditions of the experiment. In particular, the four-sided die representing the quantum probabilities at detector  $D_0$  does not yield the uniform distributions that

a fair four-sided die would. Instead, it's a dynamically “loaded” quantum die with probabilities that vary according to each of the four possible wavefunctions and the die's location in space. Rather than one number in four appearing after each toss, the number that appears depends on where the die is tossed and which of the four potential wavefunctions it could land on. It's as if the odds of a craps game played in Las Vegas were completely different from the same game played in Atlantic City.

The second four-sided idler die has some unique properties, too. Four of the 16 possible combinations of our four-sided dice metaphor had to go to zero because of the physical constraints imposed on the idler side of the experiment—specifically because of the sorting done by the quantum eraser at  $D_1$  and  $D_2$ , and because of the selective behavior of the  $D_3$  and  $D_4$  detectors. Therefore, one side of our idler die will always show a zero, but which side ( $D_1$ ,  $D_2$ ,  $D_3$ , or  $D_4$ ) depends on which row of Matrix 2 we are using.

Finally, each die is quantumly entangled with the other, but just like in a normal game of unentangled dice, the results aren't known until the last die has come to rest—regardless of the order in which that happens. The result is a very strange game of chance with two quantumly entangled dice, which is what makes this interpretation so interesting. For one, it puts to rest the counterintuitive “retrocausal” interpretations of this experiment once and for all. And for another, it implies that the experiment—and indeed the world—could very well be just another crapshoot. Sorry, Albert.

